The need of elastic RTM for elastic FWI models
Hui Huang*, Zedong Wu, Xue Xiao, Zhigang Zhang, Ping Wang (CGG)

Summary

In areas with large impedance contrasts, elastic FWI has demonstrated its advantages over acoustic FWI to sharpen velocity interfaces and improve the signal-to-noise ratio and event continuity of the corresponding FWI Image. As it is sometimes computationally prohibitive to run high-frequency elastic FWI for FWI Imaging purposes, more so for FWI Imaging gathers, RTM stack and gathers are still indispensable for model validation and reservoir interpretation. Acoustic RTM, which uses an acoustic modeling engine of wave propagation, is theoretically suboptimal to evaluate and leverage the full potential of models derived by elastic FWI using an elastic modeling engine in its inversion. In this paper, we demonstrate the necessity of elastic RTM for elastic FWI models through two field data examples.

Introduction

In the past few years, acoustic FWI (AFWI) has been successfully applied to industrial data sets to obtain velocity models for various geologic environments (Zhang et al., 2018; Wang et al., 2019). Built upon a reliable FWI, FWI Imaging, the normal derivative of the FWI velocity, has proven to offer an alternative to the most accurate migration algorithm, reverse time migration (RTM), with better illumination and higher signal-to-noise ratio (S/N) naturally, as a result of many iterations of least-squares fitting of the full wavefield data from low to high frequencies (Zhang et al., 2020; Huang et al., 2021). In areas with large impedance contrasts, such as in the presence of salt bodies, elastic FWI (EFWI) has been proposed to better handle the strong elastic effects and hence gives sharper velocity interfaces and improves S/N and event continuity of FWI Images over its AFWI counterparts (Wu et al., 2022; Ren et al., 2022).

While FWI Images often show remarkable benefits over RTM, the compute cost for high-resolution products can be prohibitively high as it needs multiple iterations for the inversion to converge at each frequency. Furthermore, generating FWI Image gathers for AVA analysis (Warner et al., 2022) would further increase the compute cost significantly. Due to these reasons, RTM still plays an important role in seismic processing, both as a convenient QC tool and further increase the three normal stresses fit.

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While FWI Images often show remarkable benefits over RTM, the compute cost for high-resolution products can be prohibitively high as it needs multiple iterations for the inversion to converge at each frequency. Furthermore, generating FWI Image gathers for AVA analysis (Warner et al., 2022) would further increase the compute cost significantly. Due to these reasons, RTM still plays an important role in seismic processing, both as a convenient QC tool and final imaging products in areas with complex geology. As more and more seismic imaging projects are moving from AFWI to EFWI, it is becoming apparent that we also need to upgrade acoustic RTM (ARTM) to elastic RTM (ERTM), which has a consistent modeling engine as EFWI and can better reap the benefits of velocity model improvements brought by EFWI.

In this study, we propose to implement ERTM that uses the same wave-propagation engine as is used in the EFWI implementation by Wu et al. (2022). Using two field data examples, we compare ERTM images with ARTM images to see if ERTM can realize more benefits of the EFWI models.

Method

To correctly evaluate and fully leverage the benefits of EFWI models, an RTM algorithm should use a wave-propagation engine consistent with that in EFWI. Following the EFWI implementation by Wu et al. (2022), our ERTM solves the elastic full wave equation built on stress vector:

\[ \frac{\partial^2 \sigma}{\partial t^2} = CD \left( \frac{\partial}{\rho} D^T \sigma \right), \]

where \( \rho \) is the density, \( \sigma \) is the stress vector defined as:

\[ \sigma = (\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{13}, \sigma_{23})^T, \]

and \( C \) is the stiffness matrix of material parameters. For an orthorhombic media, \( C \) can be written as:

\[ C = \begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix}. \]

Its entries are defined as:

\[ c_{11} = \rho (1 + 2\varepsilon_2) v_p^2, c_{22} = \rho (1 + 2\varepsilon_1) v_p^2, c_{33} = \rho v_s^2, \]

\[ c_{44} = \rho \left( \frac{1 + 2\gamma_2}{1 + 2\gamma_1} \right) v_p^2, \]

\[ c_{55} = \rho v_s^2, c_{66} = \rho (1 + 2\gamma_1) v_p^2, \]

where \( v_p \) and \( v_s \) are vertical P-wave and S-wave velocities, respectively; \( \varepsilon_1, \varepsilon_2, \gamma_1, \) and \( \gamma_2 \) are anisotropy parameters; and

\[ c_{12} = \sqrt{c_{11} - c_{66} \sqrt{(1 + 2\delta) c_{11} - c_{66} - c_{66}}}, \]

\[ c_{13} = \sqrt{c_{33} - c_{66} \sqrt{(1 + 2\delta) c_{33} - c_{55} - c_{55}}}, \]

\[ c_{23} = \sqrt{c_{33} - c_{44} \sqrt{(1 + 2\delta) c_{33} - c_{44} - c_{44}}}. \]

The derivative matrix \( D \) in Equation (1) is defined as

\[ D = \begin{pmatrix} \partial_1 & 0 & 0 & 0 & \partial_3 & \partial_2 \\ 0 & \partial_2 & 0 & \partial_3 & 0 & \partial_1 \\ 0 & 0 & \partial_3 & \partial_2 & \partial_1 & 0 \end{pmatrix}. \]

The pressure wavefield \( p \) is then calculated as the average of the three normal stresses:

\[ p = (\sigma_{11} + \sigma_{22} + \sigma_{33})/3. \]

By setting \( v_s = 0 \) everywhere, we obtain the acoustic full wave equation.

Both our ARTM and ERTM adopt a cross-correlation imaging condition on the source- and receiver-side pressure wavefields \( p_s \) and \( p_r \):

\[ I(\vec{x}) = \int p_s(\vec{x}, t) p_r(\vec{x}, t) dt. \]

The compute cost of solving an elastic wave equation is substantially increased when compared to solving an acoustic wave equation. Thus, ERTM requires much higher computational resources than ARTM. However, this extra cost of ERTM is rewarded when an EFWI velocity model is used, particularly around large impedance contrasts where the EFWI model shows considerable uplifts over the AFWI model, as we will demonstrate with two field examples.
Field data examples

The first field data example is from Walker Ridge field, GOM. It uses seismic data from two towed-streamer surveys, one full-azimuth staggered acquisition, and one wide-azimuth acquisition. The 8 Hz AFWI velocity model is obtained from an “FWI-driven velocity model building workflow” of iterative FWI and FWI-guided scenario tests; then an elastic velocity model is obtained using the 8 Hz AFWI $V_p$ model and a derived $V_S$ model from an empirical $V_p/V_S$ as the initial model for EFWI (Ren et al., 2022).

Both AFWI and EFWI models (Figures 1a and 1e) reveal complex salt geometry, indicating strong elastic effects in the study area. Given that it incorporates these elastic effects into its modeling engine during wave propagation, EFWI has the potential to produce a velocity model with a higher level of accuracy. Overall, as compared to their AFWI counterparts (Figures 1a and 1b), the EFWI model and Image (Figures 1e and 1f) exhibit less salt halo, better event continuity, and less smearing in the layering structures in sediments and basement.

We first perform 15 Hz ARTM with both AFWI and EFWI models. In contrast to the overall improvement of the EFWI model and Image over the AFWI model and Image, ARTM with the EFWI model (Figure 1g) shows only a mild quality improvement over ARTM with the AFWI model (Figure 1c). First, in the image of the ARTM with the EFWI model, the salt boundary is slightly sharper and the sedimentary layers close to the salt boundary exhibit a slightly lower noise level (blue arrows in Figures 1c and 1g). Second, the layering structure in the basement is better imaged in ARTM with the EFWI model (blue box in Figures 1c and 1g). These two observations demonstrate that the EFWI model was more accurate than the AFWI model at these locations. However, the ARTM with the EFWI model exhibits some degree of degradation around the base of salt (BOS) and in the subsalt region (yellow arrows in Figures 1c and 1g); it shows slightly less focused events and worse event continuity. We believe that the inconsistent wave-propagation engines used by ARTM and EFWI are the main cause of the image degradation.

We then perform 15 Hz ERTM with the EFWI model (Figure 1h), where an elastic modeling engine is used for both migration and velocity inversion. The ERTM image with the EFWI model outperforms the two ARTM images in Figures 1c and 1g: the salt boundary is better defined with higher clarity, and the subsalt events are better focused with improved event continuity.

These observations are validated through the analysis of RTM surface offset gathers (SOGs). The surface seismic data is partitioned into 10 offsets, ranging from 0 to 10 km with an increment of 1 km. In Figure 1d, the blue arrows highlight images of the complex salt boundaries, where we notice that the ERTM with the EFWI model exhibits a more focused event with better consistency from near to far offsets. Additionally, the yellow arrows point to images of subsalt events, where the ERTM with the EFWI model shows improved focusing and more coherent events across offsets.

We also observe that the 15 Hz ERTM image with the EFWI model is inferior to the 8 Hz EFWI Image in terms of event continuity and S/N in areas where illumination is not sufficient (blue boxes in Figure 1).

Figure 1: Walker Ridge example: (a) 8 Hz AFWI model; (b) 8 Hz AFWI Image; (c) 15 Hz ARTM image with AFWI model; (d) SOGs of the three RTM images in (c), (g), and (h); (e) 8 Hz EFWI model; (f) 8 Hz EFWI Image; (g) 15 Hz ARTM image with EFWI model; and (h) 15 Hz ERTM image with EFWI model. The location of the SOG is indicated by the red arrow in figure (c). In figure (c), (d), (g), and (h), “AE” means ARTM with AFWI model, “AE” means ARTM with EFWI model, and “EE” means ERTM with EFWI model.
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Figure 2: Another location in the Walker Ridge field with sediment-salt truncation: (a) 8 Hz AFWI model; (b) 8 Hz AFWI Image; (c) 15 Hz ARTM image with AFWI model; (d) SOGs of the three RTM images in (c), (g), and (h); (e) 8 Hz EFWI model; (f) 8 Hz EFWI Image; (g) 15 Hz ARTM image with EFWI model; and (h) 15 Hz ERTM image with EFWI model. The location of the SOG is indicated by the red arrow in figure (c). In figure (c), (d), (g), and (h), “AA” means ARTM with AFWI model, “AE” means ARTM with EFWI model, and “EE” means ERTM with EFWI model.

Figure 3: Herschel example: (a) 15 Hz EFWI model; (b) 15 Hz ARTM with EFWI model; (c) ARTM SOGs; (d) 15 Hz EFWI Image; (e) 15 Hz ERTM with EFWI model; and (f) ERTM SOGs. The locations of the SOGs are indicated by the red lines in (b). In figure (b), (c), (e), and (f), “AE” means ARTM with EFWI model, and “EE” means ERTM with EFWI model. The SOGs have two perpendicular azimuths indicated by the simple rose diagrams on top of figure (f).
We further examine another location with sedimentary layers truncating against the salt body (Figure 2). The sediment-salt truncation area is important for reservoir interpretation because it could point to possible oil and gas deposits. Compared to the AFWI model and Image (Figures 2a and 2b), the EFWI model and Image (Figures 2e and 2f) show a sharper salt flank and more distinct sediment layers truncating against the salt. Once again, ARTM fails to demonstrate a similar level of benefits of the EFWI model. To the opposite effect, ARTM with the EFWI model (Figure 2g) even reveals less continuous sedimentary layers and less distinct salt flank when compared with ARTM with the AFWI model (Figure 2c). ERTM with the EFWI model (Figure 2h) best images the truncation area and is most consistent with the EFWI Image. The uplift of ERTM with the EFWI model on the stacked image is further evident in its SOG (Figure 2d). Notably, the events in the truncation zone exhibit improved focusing and coherency from near to far offsets and display better gather flatness.

The second field data example uses an OBN data set from the Herschel field, GOM. EFWI is performed up to a frequency of 15 Hz (Wu et al., 2021). The EFWI model and Image (Figures 3a and 3d) show a well-defined salt boundary and layering sediments around it. The ERTM image with the EFWI model exhibits more concentrated sediment layers, more consistent with the EFWI Image when compared with the ARTM image with the EFWI model (yellow arrows in Figures 3b and 3e). To further assess the benefit of ERTM, we also generate RTM SOGs. The surface seismic data is separated into two perpendicular azimuths and 10 offsets from 0 to 10 km with a 1 km increment. ERTM SOGs (Figure 3f) outperform ARTM SOGs (Figure 3c) in terms of event focusing and gather flatness. This indicates inferior kinematics in ARTM and explains the dimmer subsalt amplitude on the ARTM stacked image, as shown in Figure 3b. When we compare RTM images with EFWI Image, EFWI Image reveals improved overall event continuity (blue arrows in Figures 3b, 3d, and 3e).

**Conclusion and discussion**

We have conducted ARTM and ERTM to evaluate the AFWI and EFWI models. In both field data examples, the EFWI model and Image show clear uplift over their AFWI counterparts in terms of velocity interface sharpness and image S/N and continuity. However, ARTM with the EFWI model does not show the same level of improvement over ARTM with the AFWI model. The former can be even worse in certain areas because of the inconsistent modeling engines used in ARTM and EFWI. Conversely, ERTM with the EFWI model has better event focusing and improved S/N than ARTM with the same model at places near salt complexities, e.g., truncations against the salt flank and areas beneath the salt body. However, we also need to point out that in most sedimentary basins and some locations around the salt, ARTM and ERTM may show overall comparable images when an EFWI model is used.

We note that for both examples, the salt boundary and sediment inclusions inside the salt body in the ERTM image have relatively weaker amplitudes than those in the ARTM image. This, in our opinion, is due to the conversion of a portion of the compressional energy into shear energy at sharp contrasts during elastic wave propagation; therefore, the pressure field by an elastic modeling engine shows smaller amplitudes, which leads to weaker ERTM amplitudes for the impacted events. The subsalt events in ERTM (Figure 3e) in the Herschel example show stronger amplitudes than those in ARTM (Figure 3b), because ERTM has better event focusing (Figure 3f) and the undershooting energy imaging the subsalt events is not impacted by mode conversion from the overburden salt body. On the other hand, the amplitude of EFWI Images is often observed to be relatively stronger than the acoustic counterpart because 1) FWI as an inversion approach can naturally compensate for the amplitude loss from wave propagation, and elastic wave propagation experiences a larger loss due to mode conversion; and 2) the EFWI model has overall improved kinematics compared to the AFWI model because it handles elastic effects more accurately.

We also note that the ERTM image is still inferior to the EFWI Image. We think this is mainly due to three reasons: 1) RTM uses primaries as input while FWI uses the full wavefield, which provides superior illumination over only primaries; 2) RTM adopts the Born approximation while FWI performs full-wavefield modeling, which allows FWI to compensate for transmission losses; and 3) RTM is an adjoint operator while FWI uses an iterative least-squares data-fitting process that can balance illumination and mitigate migration artifacts. Nevertheless, the imaging uplift of ERTM over ARTM makes ERTM a superior QC tool and final product in the EFWI era, especially when high-frequency EFWI Image (gathers) is unavailable or unaffordable.

**Acknowledgments**

We thank CGG for permission to publish this work and CGG Earth Data for permission to use the Walker Ridge, Gulf of Mexico and Herschel, Gulf of Mexico data examples.