Semi-inverse Relationship Between Critical Angle and Reflectivity Coefficient, And Its Implication in Seismic Survey Design, Processing, And Interpretation

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Summary

In US Gulf of Mexico (GoM), as we increasingly explore and produce from deep, clastic reservoirs that are acoustically near transparent and even hard, we are also increasingly reliant on mid and far angle ranges to image and characterize those reservoirs. However, how far can we push? The maximum angle cutoff, as a key parameter in survey design, processing, and interpretation, is unfortunately hard to define and usually selected empirically from past data and acquisition designs, which may or may not be optimal for today's challenges. This paper attempts to provide a simple theorical framework to help optimize this choice to maximize the value of seismic data acquisition and processing projects.

Methods

In survey design, processing, and interpretation of seismic imaging projects, we often come across the question as to how far shall we push the far angle to, and stack in? This question becomes more critical recently when the task is to image the deep, near-transparent clastic reservoirs. It has become ever more challenging to properly image and characterize these fields, partly because, at certain depths, the zero-incidence reflectivity between hydrocarbon or wet sand and shale becomes close to zero or even flips polarity (Gutierrez, 2018). Therefore, they are extremely difficult to image and characterize with near offset dominated seismic data. However, if these sands happen to have a desired amplitude versus offset (AVO) behavior, e.g. class III or class IV AVO, then they can be imaged more properly in the mid and far angle range as either soft or hard sands (If they stay transparent with a flat AVO even in mid and far angles, there is no meaningful approach to ever image them seismically anyway, unless velocity contrast can be resolved from transmitted waves through FWI). As the mid to far angles have a more stable polarity, focusing on mid to far sub-stacks would also help to avoid a polarity flip either from structure up-dip to down-dip, or from mid to far angles. But again, are we capturing enough far angles to form a good image and even help reservoir characterization? How far is far enough?

The maximum angle in the target reservoir is unfortunately a complex function of survey design (offset in particular), target depth, rock physics, local geology, and nearby salt geometry. Therefore, the question is difficult to answer and often we refer to existing data or past surveys for an answer ("what is the angle range for this field/reservoir?" are often taken as equivalent to "what is the angle range in this data which happen to image this field?"). Those past data and surveys may or may not be optimal for today's challenges, as the reservoirs we explored and produced from a decade ago are significantly shallower and different than those of today. However, as it is obvious that the maximum angle for the target reservoir can only be less or equal to the critical angle, we can attempt to answer this alternative question: what is the critical angle of the target reflector(s), irrespective of other factors? The answer to the second question, as shown in this work, could be less complicated and quite useful, as we can then direct our energy towards how to optimize our survey and processing to get the maximum angle as close as possible to the critical angle, within other boundary conditions.

In this section we attempt to establish a relationship between the critical angle and the zero-incidence reflectivity coefficient of a reflector. We start from the reflectivity coefficient as it is a rock physics parameter that is easy to obtain (from wells or rock physics models), and it can characterize the nature of the target reservoirs in either simple terms (weak or strong) or more quantitative manners (depth profile, etc.) for detailed modeling purposes.

For the convenience of derivation (especially for the later derivation of the critical angle), here we first consider the reflectivity coefficient of a hard reflector (incidence from a slow medium to a quick medium, positive coefficient). The zero-incidence reflectivity coefficient is:

$$R^{+} = \frac{AI^{+} - AI^{-}}{AI^{+} + AI^{-}} \tag{1}$$

where AI^{+} and AI^{-} are the acoustic impedance of the fast and slow media, respectively.

Assuming AI is a continuous function in the subsurface, we can approximate the equation above to:

$$R^{+} \approx \frac{d(AI)}{2AI} = \frac{1}{2} d(lnAI) = \frac{1}{2}\left[\ln \frac{AI^{+}}{AI^{-}}\right] \tag{2}$$

Note this is a common and quite accurate approximation in rock physics modelling. For clastic sedimentary settings, the error is usually less than 1%.

Next, for simplicity, we assume our sediments could be described by the Gardner's relationship (Gardner et al., 1974) in a clastic sedimentary setting, then the above equation could be re-written into a function of velocity contrast:

$$R^+ \approx \frac{1}{2}\left[\ln\frac{AI^+}{AI^-}\right] = \frac{1}{2}\left[\ln\frac{V_p^+\rho^+}{V_p^-\rho^-}\right] = \frac{1}{2}\left[\ln\frac{\alpha^+\left(V_p^+\right)^{1+\beta}}{\alpha^-\left(V_p^-\right)^{1+\beta}}\right] = \frac{1+\beta}{2}\left[\ln\frac{\alpha^+V_p^+}{\alpha^-V_p^-}\right] \tag{3}$$

where α^+, α^- are the Gardner coefficients of the fast and slow media and β is the Gardner exponent, as the Gardner's exponent is usually quite close across different lithologies while Gardner's coefficient could vary across lithologies.

On the other hand, based on Snell's law, the critical angle of a hard interface (incidence from a slow medium to a fast medium) is always simply a function of velocity contrast:

$$\theta_c = \arcsin\left(\frac{V_p^-}{V_p^+}\right) < 90° \tag{4}$$

Therefore, we can rewrite the above equations into:

$$\theta_c \approx \arcsin\left(\frac{\alpha^+}{\alpha^-}e^{-\frac{2R^+}{1+\beta}}\right) = \arcsin\left(Be^{-1.6R^+}\right) \tag{5}$$

where $B = \frac{\alpha^+}{\alpha^-}$ is a Gardner coefficient ratio between the fast and slow media (shale and sand, for example), that usually varies between 1.0 to 1.1 depending on the basin setting. Physically, B value quantifies the ratio of density between fast and slow media if they have the same velocities therefore it is tightly bounded by the physical law of packing and compaction of mineral grains. B value can be easily calibrated using wells.

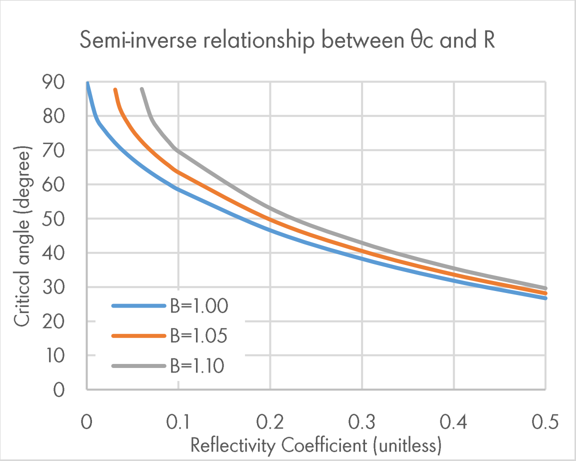

Figure 1 A quantitative semi-inverse relationship between critical angle and the zero-incidence reflectivity coefficient assuming B=1. Note there is a drastic increase of critical angle for weak reflectors.

Equation (5) is surprisingly a simple equation to describe the relationship between critical angle and reflectivity. Essentially, this equation indicates there is a semi-inverse relationship between θ_c and reflectivity coefficient: stronger reflectors have less theoretical angle range. Figure 1 visualizes this relationship with different B values found in GoM.

From Figure 1, it can be observed that a reasonable estimation of critical angle could still be achieved even without any knowledge of B. For example, when $R^+ = 0.10$, the critical angle range is 60~63~69 degree, depending on the exact B value. This level of estimation could be good enough as our one-sided error is just 1~2 angle bins assuming we bin every 3 degree into a trace. This is usually good enough already for survey design and processing parameterization discussions.

The above relationship could be easily validated by rock physics modelling based on field velocity and density trends. The procedure is such, first we compute critical angle using Snell's law based on local velocity trends of sand and shale; next we calculate the reflectivity coefficients of sand-shale interface based on rock physics modeling. Finally, we compute the critical angle as a function of reflectivity coefficient based on equation (5) and compare to the results from step 1. This process is repeated four times with rock physics trends in GoM field 1 and field 2, each with both oil-filled and wet cases. Both fields have a same B factor of 1.068. The result of this comparison is shown in Figure 2. The proposed approximation generally yields satisfactory results (within 3 degree) compared to the truth, except when the expected critical angle is approaching 90 degree.

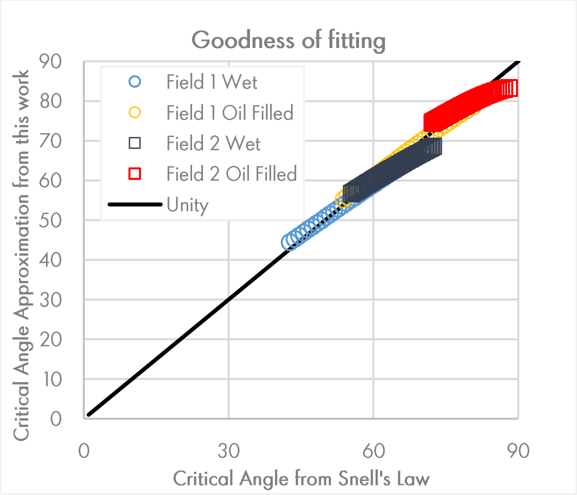

Figure 2: Goodness of fitting of critical angle computed from the proposed approximation and the theoretical ground truth using Snell's law, based on rock physics modelling.

Equation (5) could also be verified using either synthetic gather of well logs, or by checking migrated seismic gather in open basin areas from long offset seismic survey, provided the migration did not set an explicit or implicit angle limit (via offset cutoff). In surveys where offset is limited (<10 km), the angle range observed in migrated data are usually limited by survey offset and therefore does not reflect its actual limit. In two US GoM fields, the derived relationship provided good estimations of angle ranges of multiple stacked target reservoirs that fits both modeling and data observations.

For soft reflectors, there is no critical angle limitation on the interface as the incidence ray path is from a fast medium into a slow medium (although there will be a limiting angle for reflectors below which will be discussed in the discussion section). However, the same rule demonstrated above will still govern the angle range even for soft reflectors. First, subsurface sediments will create symmetric or near-symmetric lithology interfaces with opposite reflectivity, for example, pairs of sand top and sand base reflectors. Secondly, as the seismic energy travels two-way from surface to subsurface, and then back to surface, theoretically the same ray will pass the same (or equivalent) reflector twice with opposite reflectivity, and the soft reflector for a down-going wave will become a hard reflector for the up-going wave, although not at the same location. Due to these two reasons, the critical angle is always simply a function of the velocity contrast between interfacing layers, regardless of polarity.

Discussion

The relationship established above has some profound implications in seismic survey design, processing, and seismic interpretation stages.

In the Gulf of Mexico, as we deplete shallower, amplitude-supported reservoirs, we are increasingly exploring and producing from deeper, more complex, near transparent reservoirs today. To be able to effectively stack, image and interpret these reservoirs, we need to acquire enough traces in the mid and far angle ranges. We can add fold by either adding data acquisition density (nodes, shots), or by adding offset or angle range, which, in turn, requires us to be explicitly intentional in the attainable angle range for the target reservoirs.

In survey design exercises (raytracing, De-remigration, etc.,), critical angle, or maximum angle is usually not an output, but an input to the design workflows as those workflows does not explicitly model rock physics. Therefore, we have to obtain the knowledge of critical angle or maximum angle cutoff beforehand. For instance, Figure 1 shows that weaker reflectors, which appear very dim in the nears, could have substantial angle range to help image them better in the mid and far. It is very surprising that, if the reflectivity coefficient is less than 0.05 (absolute value), the critical angle would open drastically, all the way to 90 degrees if the reflectivity coefficient drops further. Knowing the critical angle of the target level would allow us to make choices in optimizing survey design by adding data density or adding survey offset, which will have very different cost profiles (and different benefits other than traces as well). When the critical angle is limited, perhaps the only choice is to increase data density, whereas when reflectors are weak, we could incrementally increase offset to ensure inclusion of the very far angles if they are important or more economic.

Figure 3 reflects the subsurface illumination for a target event for an ocean bottom node (OBN) survey designed with an explicit emphasis on the desired angle range needed for imaging near transparent reservoirs in a cost-effective way. Intuitively we know there is a correlation between offset and angle acquired. However, it is important to quantitatively capture this relationship for decision making and cost engineering. In this survey design, knowing the critical angle ranges for target reservoir as function of depth and lithology allow the practitioner to focus on imaging the weak reflector better in the mid to ultra-far angle ranges by optimizing the cross-line offset and the corresponding node area and shot rind.

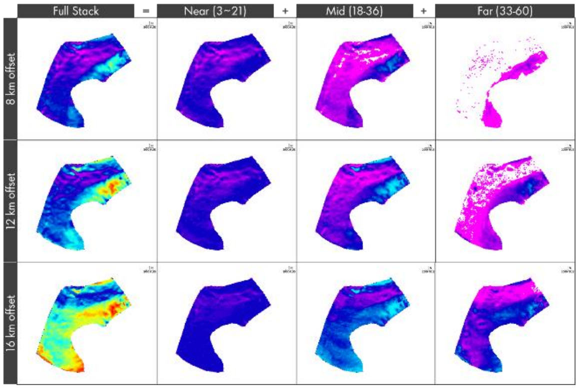

Figure 3 Hit count maps of different survey design options (rows of different offsets) and the resulting hit count in full stack and different angle range (columns). Bright color indicates more hit count. The hit count maps are prepared with ray-tracing methods but was substantiated with de-remigration and data migrations. At least 12km offset to image the target in the mid and far angle range.

The awareness of an attainable angle range is also critical in the processing and interpretation stages. For seismic surveys targeting deep, near-transparent clastic reservoirs, it is important to use the relationship demonstrated in Figure 3 to optimize processing parameters such as offset cutoff, angle muting limits etc. In practice, angles above 50 degrees are rarely processed, or interpreted even if they had been acquired, without such explicit awareness. Sometimes we even cut to below 40 degree as we set our angle range expectations based on bright, amplitude supported reservoirs which happen to have limited angle range. Sometimes there are hard-coded angle cutoffs even in migration algorithms. This could imply leaving money (data) on the table and potentially hydrocarbons left in the ground.

It is useful to point out that the critical angle is only a theoretical limit of the reservoir we target. Whether we can reach that angle is dependent on many factors we can control, as mentioned above, as well as factors we can never control. If the survey design allows, the incidence angle will gradually widen the deeper the wavefronts travel in sediments. That means for many reservoirs that are too shallow to allow the incidence angle to build up, it might be impossible to reach their critical angle. The benefits of opening offset to allow farther angles, mostly goes to the deeper sediments where we happen to have imaging problems. The other exceptions here are for reflectors that are directly beneath a very strong reflector (salt base, shale-carbonate). The angle range of the deeper reflectors will be limited by the narrow angle range imposed by the strong reflector above. This salt or strong reflector limited angle maximum (SLAM) could be easily calculated and used in a similar way as how critical angles could be used as discussed in this paper.

Recently, the revolution of full waveform inversion (FWI) has helped tremendously on focusing the extreme far angle sub-stack images by providing great accuracies in the overall velocity model, but we are still early in the journey. Additionally, the same long offset low frequency surveys that enable FWI have also brought us the extreme far angles as a by-product in many multi-client seismic campaigns in the GoM. However, it is important to mindfully keep the long offset data during imaging processing to preserve the far angle ranges discussed in this paper to fully capture potential values of those surveys.

Finally, although we may acquire, process, and interpret near-transparent sediments within the calculated critical angle range, we should still be cautious on conducting quantitative interpretation and AVO analysis using the ultra far angle range before and even after carefully QC and calibrate the data with well synthetics. Extremely far angle (45+) data may contain more contaminations from anisotropy, far field velocity model errors, mode conversion or interbed multiples, etc.. Nonetheless, they could be tremendous help in imaging some of the deep, hard to see reflectors.

Conclusion

This work has shown that there is a simple relationship between zero-incidence reflectivity coefficient and critical angle:

$$\theta_c \approx \arcsin\left(Be^{-1.6R^+}\right)$$

The only calibration parameter $B = \frac{\alpha^+}{\alpha^-}$ is the Gardner coefficient ratio between the fast and slow media which is easy to calibrate with well data and quite stable across different fields in the GoM. Even if without calibration of B value, the range of critical angle could still be reasonably estimated as the error band of critical angle from the proposed approximation is rather tight.

Utilization of this simple relationship will enable us to optimize seismic projects throughout survey design, processing, and interpretation stage, to image the near transparent sedimentary environments better with longer offset surveys that opens the angle range within critical angle and other boundary conditions.

When reflectivity is weak, there might be room to stack farther!

Acknowledgment

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