An improved holo-hilbert spectral analysis guided by fast matching pursuit

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Summary

Seismic signals are non-stationary and nonlinear affected by excitation circumstances, absorption and attenuation of the stratum, and characteristics of the receiving instruments. Time-frequency spectral analysis methods are effective and widely used in describing the time-varying features of seismic signals. Nevertheless, these methods are all based on additive expansions, and cannot represent the nonlinearity of signal, which is achieved through a multiplicative process. Therefore, the information provided by time-frequency spectrum is incomplete. In order to quantify the nonlinearity of seismic signal, that is, the modulation effect of low-frequency components on high-frequency components, the improved Holo-Hilbert spectral analysis method is introduced in this paper. The improved Holo-Hilbert spectral analysis (HHSA) contains nested variational modal based fast matching pursuit decomposition and Hilbert transform to achieve a full informational spectrum represented by two frequency dimensions. One dimension represents frequency-modulated (FM) frequency, which characterizes the non-stationarity of signal, and the other dimension represents the amplitude-modulated (AM) frequency, which indicates the nonlinear effect in signal. Synthetic and field data examples verify the feasibility of the improved HHSA in characterizing the cross-frequency interaction of non-stationary seismic signals and predicting reservoirs.

Introduction

Seismic wavelets propagate in underground media, and low-frequency components modulate high-frequency components to form seismic signals. The energy attenuation of high-frequency components indicates the existence of reservoirs. There are two types of modulation in seismic signals, one is frequency modulation, which represents non-stationary processes, and the other is amplitude modulation, which represents nonlinear effects. Traditional time-frequency analysis methods (STFT, CWT, ST, MP, and HHT) are based on additive expansion and use time-frequency spectra to reveal the non-stationary processes of signals. However, amplitude modulation is often a multiplication process that represents the nonlinear effects of the signal. Existing methods cannot characterize the modulation effects in non-stationary seismic signals and lack consideration for their nonlinear characteristics.

In recent years, Huang (Huang,2016) introduced holo-hilbert spectral analysis as a method to comprehensively characterize the data present in nonlinear and nonstationary signals. By converting the time-frequency-energy spectrum to the frequency-frequency-energy spectrum, this method compensates for the shortcoming of conventional time frequency analysis. As the evolution and development of HHT, HHSA is composed of nesting Hilbert transform and empirical mode decomposition. Since its proposal, this method has been widely applied in engineering fields, but has not been applied to seismic signal analysis.

Moreover, traditional empirical mode decomposition algorithm is not adaptive when determining the number of mode decomposition. Therefore, this paper introduces a variational mode decomposition algorithm optimized based on matching pursuit algorithm(Wang,2024), which is nested into an improved HHSA algorithm to make it more adaptive and anti-noise. The improved HHSA algorithm introduces the study of nonlinear amplitude modulation in seismic signals, elevates the time-spectrum analysis of seismic signals to a higher dimension.

Holo-Hilbert Spectral Analysis

Holo-Hilbert spectral analysis offers a comprehensive and informative spectrum displayed in a two-dimensional frequency representation. In the Holo-Hilbert spectrum analysis (HHSA), it is possible to analyse both the carrier frequencies and the amplitude modulation frequencies of the signal at the same time. HHSA utilises a combination of nested empirical mode decomposition and Hilbert transform to provide a comprehensive informational spectrum that is represented by two frequency dimensions.

EMD is a data-driven approach to decompose the signal into several intrinsic mode functions (IMFs) without the selection of band-pass filter cut-offs.

In general, the analysis flow was conducted as follows:

（1）The original signal $s(t)$ is decomposed into several IMFs by using EMD, with these known as the first layer IMFs, and expressed as follows:

$$s(t)=\sum_{j=1}^{n}c_j(t)+r_n=\sum_{j=1}^{n}a_j(t)\cos\theta_j(t)+r_n \quad (1)$$

where $\mathrm{Re}\{\ \}$ represents taking real part. $c_j(t)$ is the j^{th} $\mathrm{IMF_{FM}}$ component, which is amplitude-modulated and frequency-modulated. $a_j(t)$ and $\omega_j(t)$ are the instantaneous amplitude and instantaneous frequency of the j^{th} $\mathrm{IMF_{FM}}$ at time t, respectively. $\cos\theta_j(t)$ is the first layer phase function.

（2）The instantaneous frequency obtained from the first layer EMD is then applied by the Hilbert-Huang transform (HHT).

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$$s(t) = \mathrm{Re}\left\{ \sum_{j=1}^{n} a_j(t) e^{i\int_t \omega_j(\tau)d\tau} \right\} + r_n \quad (2)$$

$\cos\int_t \omega_j(\tau)d\tau$ is called the "carrier wave ", and it is presented along the X-axis in the Holo-Hilbert spectrum.

(3) Take the absolute value of each IMF_{FM}, and find all the local maxima of the absolute-valued function of IMF_{FM}. Construct the envelope function $\hat{a}_j(t)$ by a natural spline through all the maxima.

(4) The second layer EMD is obtained by applying the EMD to the amplitude function $\hat{a}_j(t)$, given as:

$$\hat{a}_j(t) = \sum_{k=1}^{m} c_{jk}(t) + R_{jm} = \sum_{k=1}^{m} a_{jk}(t)\cos\Theta_{jk}(t) + R_{jm} \quad (3)$$

where $a_{j,k}(t)$ is the instantaneous amplitude of the k^{th} amplitude-modulated intrinsic mode function (IMF_{AM}) decomposed from the j^{th} IMF_{FM}, respectively, at time t. $\cos\Theta_{jk}(t)$ is the second layer phase function and R_{jm} is the residual after the second-layer decomposition.

Thus, the whole expansion of two-layer EMD can be expressed as:

$$s(t) = \sum_{j=1}^{n}\left[\sum_{k=1}^{m} a_{jk}(t)\cos\Theta_{jk}(t) + R_{jm} \right]\cos\theta_j(t) + r_n \quad (4)$$

Equation (4) reveals the multiplicative process corresponding to amplitude modulation, which characterizes the nonlinearity of signals. The HHT is applied to these IMFs to determine the instantaneous frequency and amplitude of amplitude modulation. The instantaneous frequency and amplitude of this two-layer IMF was projected to space to obtain the three-dimensional Holo-Hilbert Spectrum which describes a complete power spectrum of cross-frequency dynamics varied with time series.

Proposed method

In this paper, the nested empirical mode decomposition algorithm of HHSA is improved by variational modal based fast matching pursuit decomposition. Matching pursuit (MP) was proposed by Mallat(1993), which is widely used in various fields of seismic processing and interpretation. The fast matching pursuit (FMP) algorithm (Fu, 2022) introduces cross-correlation threshold and multi-atom extraction technology to dynamically reduce dictionary redundancy based on the traditional matching pursuit algorithm.

Variational mode decomposition (VMD) was proposed by Dragomiretskiy (2014) and then was applied in seismic data analysis. Compared to traditional empirical mode decomposition (EMD), VMD has stronger noise robustness and a more solid mathematical theoretical foundation.

Firstly, The signal $s(t)$ is decomposed by VMD**.** And the decomposition is expressed as:

$$s(t) = \sum_{q=1}^{q} u_q(t) \quad (5)$$

where t denotes time. $u_q(t)$ is the q th component of the intrinsic mode function decomposed by VMD.

In FMP, a redundant time-frequency atom library is constructed. By expanding, translating, and modulating a signal using the window function (Ricker or Morlet wavelet), we can obtain an over-complete dictionary. Equation (6) and (7) are the expression of the Gabor and the Morlet time-frequency atom respectively. We define the index γ of the dictionary $D = \{g_\gamma(t)\}$, which contains three parameters u, ς, φ,

$$g_\gamma(t) = \left(\frac{1}{\pi\alpha^2}\right) 1/4 e^{-\frac{(t-\mu)^2}{2\alpha^2}} e^{i2\pi\zeta(t-\mu)} \quad (6)$$

$$m_\gamma(t) = \left(\frac{2\ln 2}{\pi}\right)\sqrt{\zeta} e^{-\ln 2\zeta^2(t-\mu)^2} e^{i[2\pi\zeta(t-\mu)+\varphi]} \quad (7)$$

The precise representation of the intrinsic mode function is expressed by fast matching pursuit as:

$$u_q(t) = \sum_{i=0}^{p-1} \langle R^i s, g_{\gamma i} \rangle g_{\gamma i} + R^p s \quad (8)$$

where $g_{\gamma_p}(t)$ is the best matching atom after p th iteration. $\langle \cdot \rangle$ is the inner product, $R^p s$ is the residual vector after p th iteration. This paper replaces the variational modal based fast matching pursuit decomposition with EMD in the HHSA. The Flow chart of the improved holo-hilbert spectral analysis method is as follows.

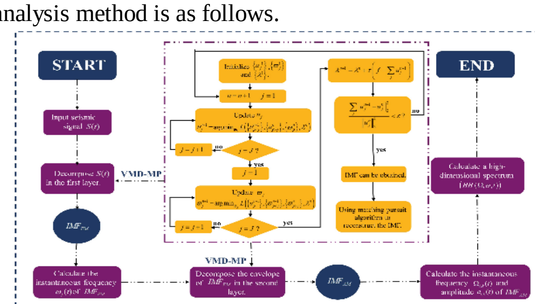
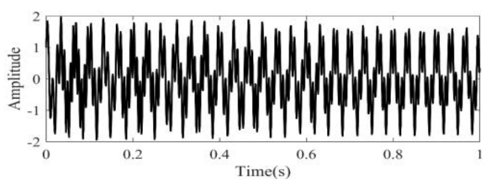


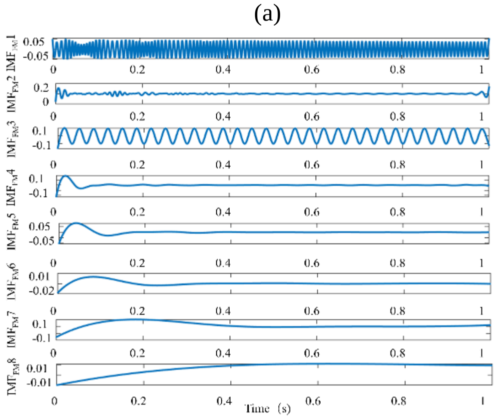
Figure. 1. Flow chart of the improved holo-hilbert spectral analysis method.

Model tests

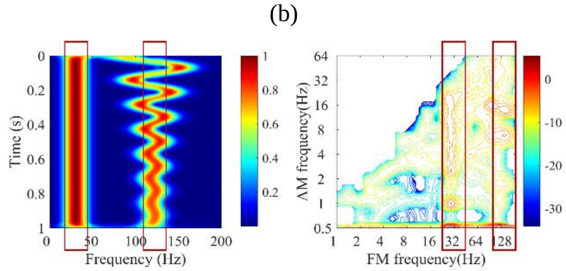
To verify the feasibility of the improved Holo-Hilbert spectral analysis method in analyzing nonlinear and non-stationary signal, we designed two test signals in figure 2 to measure. Figure 2a is an amplitude-modulation signal, and Figure 2b shows the IMF_{FM} obtained by variational modal based fast matching pursuit decomposition in the first layer. Figure 2c shows the time-frequency spectrum obtained by the short time Fourier transform and Holo-Hilbert spectrum obtained by the Holo-Hilbert spectral analysis respectively. The short-time Fourier transform can only characterize the non-stationarity of the original signal by describing the change of frequency over time, however, it lacks the ability to quantify the nonlinearity effect. In the Holo-Hilbert spectrum, not only the frequency components contained in the signal can be seen, especially the 30 Hz and 120 Hz frequency components with abundant energy distribution on the FM frequency axis, but also all the modulation effects of low-frequency components on the high-frequency components can be seen completely and clearly.



(a)

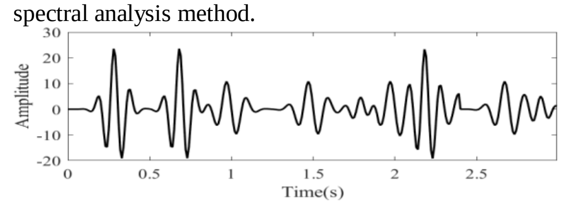


(b)

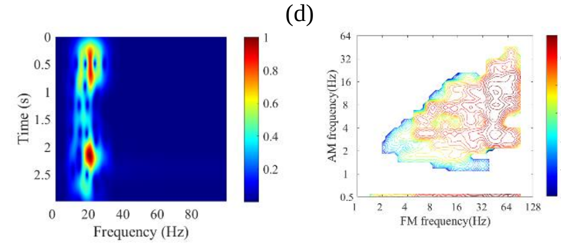


(c)

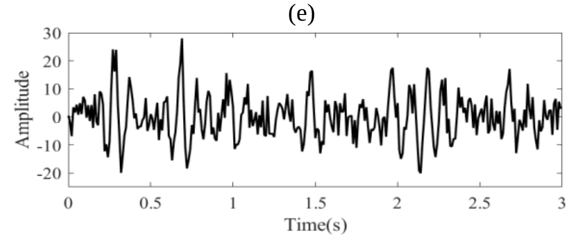
As illustrated in Figure 2d, the synthetic signal is made up of seven Morlet wavelets with frequencies of 10, 20, 30, and 35 Hz and various center times. The signal lasts 300ms and the noisy synthetic signal is shown in Figure 2f. Figures 2e shows the spectrum obtained by the short time Fourier transform and HHSA respectively. Figure 2g shows the spectra of original synthetic noisy signal obtained by the short time Fourier transform and HHSA respectively. Since the frequency component of the noisy signal is usually high frequency, it can be observed that the recognition of high frequency noise by traditional STFT is very weak and difficult to identify. Based on this result, it can be seen that HHSA is very sensitive to the recognition of high-frequency noise, which is also a significant advantage of Holo-Hilbert spectral analysis method.



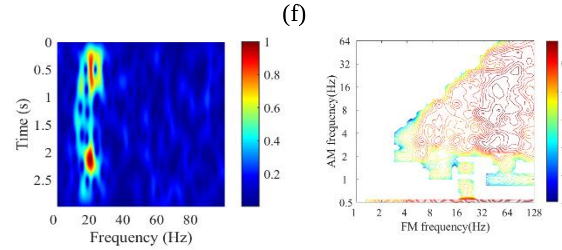
(d)



(e)



(f)



(g)

Figure. 2 Model test for comparing Spectra results.
(a) signal 1: a synthetic amplitude-modulation signal, (b) IMF_{FM} obtained by variational modal based fast matching pursuit decomposition, (c) Time-frequency spectrum obtained by STFT and improved HHSA, (e) signal 2: the synthetic seismic signal, (f) Synthetic noisy signal. (g) Time-frequency spectrum obtained by STFT and improved HHSA of noisy signal.

Field Data Application Case

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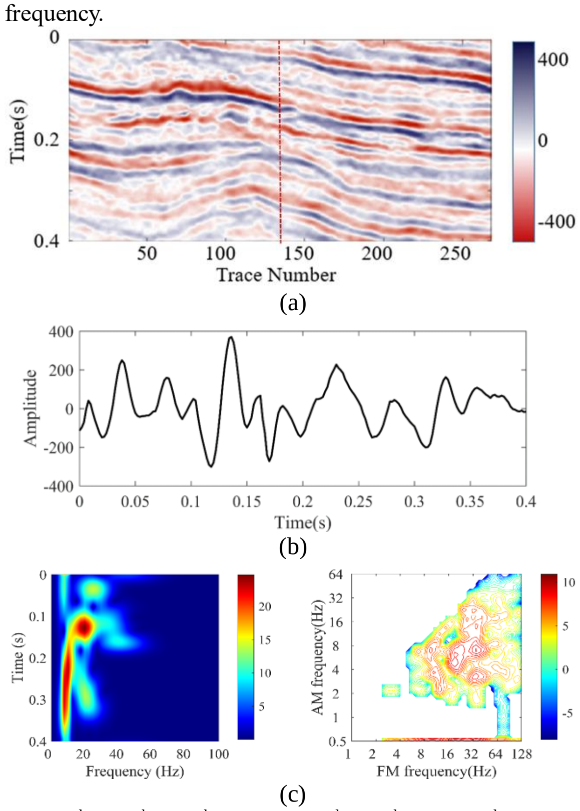
Further, the improved Holo-Hilbert spectral analysis method is applied to a field data. As shown in Figure 3a, the seismic data contains 268 traces with 201 sampling points per trace. The time sampling interval is 0.002 s. We extract the 131st seismic data and perform spectral analysis on it in Figure 3b. Figure 2e shows the spectrum obtained by the short time Fourier transform and the improved HHSA respectively. It can be observed that the energy of high-frequency components in the time-frequency spectrum obtained by the short-time Fourier transform is very weak and difficult to identify. But in the corresponding Holo-Hilbert spectrum, high-frequency components are presented through isolines with high energy density, which is also a significant advantage of Holo-Hilbert spectral analysis method. In addition, the Holo-Hilbert spectrum uses the extra dimension of AM frequency to quantify the modulation effect existing in each seismic signal, which reduces the singleness of seismic signal characterization only with FM frequency.



The 50th, 100th, 130th, 131st, 132nd, 160th and 220th seismic traces are selected from this seismic section, and the Holo-Hilbert spectral analysis method is carried out on these seven seismic signals to observe the changes of Holo-Hilbert spectrum of reservoir and non-reservoir. Figure 2d shows 3D view of the Holo-Hilbert spectra of seven seismic traces. The 130th, 131st and 132nd seismic traces represent the seismic signals through which the reservoir passes. Compared with the other four seismic signals that do not pass through the reservoir, it can be found that not only the corresponding time-frequency spectra show attenuation characteristics of high-frequency components, but also the high-frequency components of the FM frequency represented by the horizontal axis in the Holo-Hilbert spectra also attenuate, and the range of the AM frequency represented by the vertical axis hardly changes. This is because the AM frequency corresponds to the low-frequency component that changes slowly. In conclusion, Holo-Hilbert spectral analysis can be used as an auxiliary means, and combined with time-frequency spectral analysis and other technologies, it has certain application potential in the reservoir prediction with attenuation characteristics.

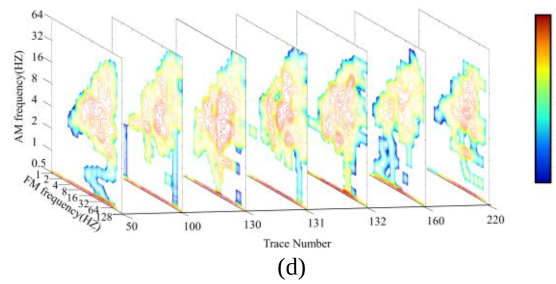


Figure. 3 Field seismic profile for comparing Spectra results.
(a) field seismic profile (b) The 131st seismic trace.(c)Time-frequency spectrum obtained by STFT and improved HHSA,(d)Three-dimensional view of the Holo-Hilbert spectra of seven seismic traces.

Conclusion

An improved holo-hilbert spectral analysis guided by fast matching pursuit is proposed in this study. The existing time-frequency spectral analysis methods reveal the non-stationarity of signal through the two dimensions of time and frequency. The improved Holo-Hilbert spectral analysis method consists of nested variational modal based fast matching pursuit decomposition and HHT. It transforms the traditional time-frequency-energy spectrum into the frequency-frequency-energy spectrum, which completely presents the modulation effect generated by all nonlinear processes and the non-stationarity of the signal. This method has a huge potential for geophysical applications especially on seismic interpretation.

Acknowledgments

We would like to acknowledge the sponsorship of National Natural Science Foundation of China (42174139, 41974119, 42030103) and Science Foundation from Innovation and Technology Support Program for Young Scientists in Colleges of Shandong province and Ministry of Science and Technology of China.