Local uncertainty quantification for 3D high resolution subsurface characterization based on data assimilation: an Ensemble Transform Kalman filter Full Waveform Inversion strategy

Alexandre Hoffmann, Univ. Grenoble Alpes, France, Romain Brossier, Univ. Grenoble Alpes, Ludovic Métivier, Univ. Grenoble Alpes/CNRS, France, and Alizia Tarayoun, Univ. Grenoble Alpes, France*

Full waveform inversion (FWI) is a high resolution seismic imaging method based on the iterative minimization of the misfit between observed and synthetic data. The synthetic data is obtained through the solution of partial differential equations representing the wave propagation within the subsurface. Designed at the beginning of the 80s (Lailly, 1983; Tarantola, 1984), FWI was first applied to 3D field data by Sirgue et al. (2010), producing an unprecedentedly high resolution estimation of the velocity model. FWI is now routinely applied in the hydrocarbon exploration industry and is an essential step of the seismic imaging workflow. More and more, thanks to the recent increase of computing capabilities, FWI output models are directly used to derive reflectivity images, bypassing the conventional migration step (Huang et al., 2021). They can also contribute to the geological interpretation as FWI yields "quantitative" estimates of the velocity models.

This common practice of FWI tends to forget a key aspect of the method: from a mathematical standpoint, FWI is an ill-posed inverse problem, whose solution is in essence non-unique. Basing imaging results and their geological interpretation on a single output FWI model can thus be dangerous, and there is a need for methods being able to quantify the uncertainty attached to a FWI model. Such methods can be classified into two main families: global and local ones. Global schemes rely on a global sampling of the posterior Probability Density Function (PDF) through typically Markov Chain strategies, under a Bayesian formalism (see Gebraad et al., 2020, for an example). While such schemes are highly desirable in their ability to provide a general view of the uncertainty attached to the FWI solutions, the corresponding computational cost make them difficult to apply to realistic scale problems. On the other hand, local uncertainty quantification schemes are based on an approximation of the posterior covariance operator in the final output model (i.e. Mulder and Kuvshinov, 2023). Such local schemes quantify the variability of the solution with respect to a given FWI result instead of an ideal "ground truth", making the uncertainty quantification problem easier while providing valuable insights in the FWI results.

Such local schemes conventionally rely on a low-rank approximation of the inverse Hessian operator through (potentially randomized) truncated Singular Value Decomposition (SVD). These methods require to sequentially compute Hessian-vector products, which amounts to the solution of several 3D wave propagation problems. They also need to specify a threshold for the truncation which is always arbitrary. In this work, we develop an alternative to these schemes based on an Ensemble Transform Kalman Filter (ETKF), a method developed in the field of data assimilation with applications in weather forecasting (Evensen, 1994; Thurin et al., 2019).

Starting with an initial FWI model, we generate an ensemble of models by adding to it zero mean random perturbations. The perturbations are spatially correlated to make sure the spatial wavenumber content of the perturbed models is compatible with the considered frequency band. Then, we decompose the available data in subsets of data. This decomposition can be done following a conventional hierarchy strategy (frequency continuation, time-offset windowing) or using for instance random shot subsampling, which we adopt in this study.

FWI is ran using each member of the ensemble as a starting model. After the inversion of each subset of data, a statistical correction is performed on each model: the analysis step in ETKF. This corrects the current ensemble to bring each member closer from the data of the next subset, while mitigating the deviation to the current mean model. The process ends when all the data has been used. In Figure 1 we present the application of this ETKF-FWI scheme to 3D field data from the North Sea. We show a vertical slice of the estimated mean model and its associated variance computed for 3 ensemble sizes: 10, 50, and 200 models. While using 10 models leads to undersample the variance, the variance extracted at a single pixel shows that with 50 and 200 members we converge towards a Gaussian distribution, which comforts us in the interpretation of the results and the reliability of this local uncertainty analysis scheme.

Compared with conventional SVD based methods, our combined ETKF-FWI scheme is intrinsically parallel as each FWI process can be ran independently. The algebraic operations involved at each analysis step involve only low rank operators and are thus negligible in terms of computation cost. This local uncertainty analysis scheme is thus particularly suited to run in parallel on large scale computing clusters, such as the forthcoming exascale machines. Implementing our ETKF-FWI on such a computing device is part of ongoing developments based on this work.

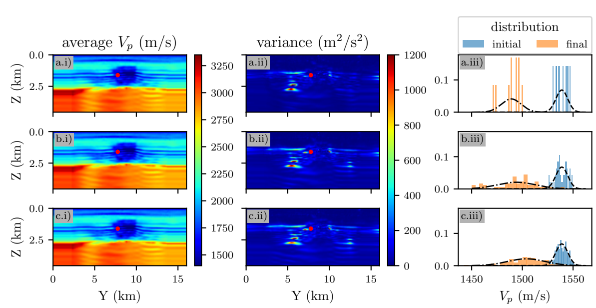

Figure 1: (a.i-c.i) Inline vertical section of the final mean V_p model, (a.ii-c.ii) final variance, (a.iii-c.iii) distribution of the models at a given point marked in red. The results are obtained with three ensemble sizes (a) $N_e = 10$, (b) $N_e = 50$, (c) $N_e = 200$.