# Least-squares Migration utilizing high-resolution Point Spread Functions generated from Local Angle Domain illumination vectors

*Matan Shustak*, Orly Kletenik-Edelman, Ronit Levy and Zvi Koren, AspenTech*


## Summary

Structural/stratigraphic seismic images, obtained by seismic migrations applied in complex geological areas with restricted acquisition coverage and frequency band, are normally blurred, containing migration artifacts that limit the interpretation of target geological features. Data and Image-domains Least-Squares Migrations (LSM) (e.g., Fletcher et al., 2016) are routinely applied to overcome these challenges, yielding enhanced (de-blurred) images. Image-domain LSM methods are more efficient than data-domain methods, and they are normally based on the application of Point Spread Functions (PSFs) (e.g., Lecomte, 2008) computed on sparse spatial locations (avoiding interference among the PSFs). This work proposes an innovative PSF-driven LSM method based on the local angle domain (LAD) imaging system (Koren and Ravve, 2011). It inherently provides the (available) rich in-situ incident/scattered (I/S) illumination vectors which are further used to construct high-resolution PSF operators at each image grid point. The PSFs are then used as deconvolution operators to enhance the migration image. We demonstrate the imaging enhancements provided by the proposed LAD-PSF LSM approach on real data.


## Introduction

LSM is routinely applied in seismic imaging to enhance obtained seismic reflectivity image quality. It can be considered as an image de-blurring method that addresses incomplete or sparse illumination of the in-situ I/S wavenumber vectors, effectively reducing seismic imaging artifacts and enhancing the frequency band.

PSF-based LSM involves deconvolution challenges, where the PSFs serve as the imaging system's impulse responses or blurring operators. When the PSF is well-defined, the convolution process can be reliably inverted (deconvolved) to achieve high-resolution images.

The LAD-based imaging system is based on a unique (bottom-up ray-based) mapping of the recorded seismic data events from the acquisition system into the image grid points, where the imaged events associated with each point are decomposed (binned) into high-resolution 4D LAD tables consisting of their corresponding I/S directional (dip/azimuth) and opening angle/azimuth bins (Figure 1). The illumination power of each LAD bin is computed as well and is used for construction of the PSFs. Their application on the migrated events considerably improves amplitude preservation of the reflected/diffracted images. Compared to other LSM methods, the proposed one is very efficient, making it possible to apply the PSF deconvolution operations post-migration, along the fine grid image points.

The imaging enhancement obtained by the proposed LAD-PSF LSM method is demonstrated using the Volve field OBC dataset and a Southern North Sea dataset.

## Method

LSM in the time domain, under the assumption of a linear modeling operator, can be defined with the data fitting cost function,

$$\Phi(\mathbf{m}) = \frac{1}{2}\|\mathbf{d}_{mod} - \mathbf{d}_{obs}\|^2 = \frac{1}{2}\|\boldsymbol{L}\mathbf{m} - \mathbf{d}_{obs}\|^2 \ , \quad (1)$$

where $\mathbf{m}$ is the reflectivity model vector, $\mathbf{d}_{mod}$ is the modeled data vector, $\mathbf{d}_{obs}$ is the observed data vector, and $\boldsymbol{L}$ is the forward modeling operator. The least squares solution of equation 1 is referred to as the image-domain LSM,

$$\widehat{\mathbf{m}} = (\boldsymbol{L}^*\boldsymbol{L})^{-1}\boldsymbol{L}^*\mathbf{d}_{obs} = \boldsymbol{H}^{-1}\mathbf{m}_{mig} \, , \quad (2)$$

where, $\boldsymbol{L}^* \approx \boldsymbol{L}^{\boldsymbol{T}}$ is the adjoint state operator, $\boldsymbol{L}^*\mathbf{d}_{obs} = \mathbf{m}_{mig}$ is the migrated image, $\boldsymbol{H} = \boldsymbol{L}^*\boldsymbol{L}$ is the approximated Hessian matrix, and $\widehat{\mathbf{m}} = \mathbf{m}_{mig} + \delta\mathbf{m}$ is the enhanced (de-blurred) reflectivity. The image domain cost function can now be written as,

$$\Theta(\widehat{\mathbf{m}}) = \frac{1}{2}\left\|\boldsymbol{H}\widehat{\mathbf{m}} - \mathbf{m}_{mig}\right\|^2 \ . \quad (3)$$

The approximated Hessian $\boldsymbol{H}$ , referred to also as the imaging system impulse response or the blurring operator, serves as the PSF operator.
Amplitude preserved high-frequency asymptotic integral migrations require, in addition to computation of the asymptotic Green's functions (two-way traveltimes and complex amplitudes), the gradient vectors of the two-way traveltimes $\tau_{IS}$ at the image points,

$$\boldsymbol{\nabla}\tau_{IS}(\mathbf{x}) = \mathbf{p}_{IS}(\mathbf{x}) = \mathbf{p}_I(\mathbf{x}) + \mathbf{p}_S(\mathbf{x}) \ , \quad (4)$$

where both incident (source-related) and scattered (receiver-related) slowness vectors, $\mathbf{p}_I(\mathbf{x})$ and $\mathbf{p}_S(\mathbf{x})$, are considered to be pointing into the image location, and the vector $\mathbf{p}_{IS}(\mathbf{x})$ is the I/S illumination-slowness vector at image point $\mathbf{x}$. This information is inherently computed in the LAD migration (Figure 1). The I/S illumination-slowness vectors for each LAD bin can be written as $\mathbf{p}_{IS} = \mathbf{p}_{\nu\gamma}$ , where $\nu$

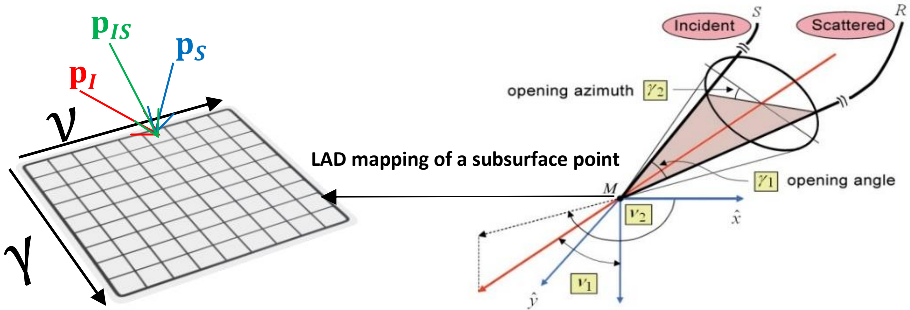


*Figure 1 demonstrates the subsurface LAD decomposition and direct application of the I/S- illumination slowness vectors for PSF calculation at individual image points.*

and $\gamma$ indicate the I/S illumination-slowness direction (dip/azimuth) and opening angle/azimuth, respectively. The corresponding wavenumber is given by,

$$\mathbf{k}_{\nu\gamma} = \omega \mathbf{p}_{\nu\gamma} \ , \quad (5)$$

where $\omega$ is the angular frequency.

The total I/S directional illumination power $I_\nu = \sum_\gamma I_{\nu\gamma}$ is then computed. Next, the estimated spectrum of the source functions $S(\omega)$ is mapped into the wavenumber domain $S(\mathbf{k}_\nu)$ , and the wavenumber domain PSF is then constructed as

$$H(\mathbf{k}_\nu) = I_\nu \, S(\mathbf{k}_\nu) \ . \quad (6)$$

The LAD-PSF LSM operation defined in equation 3 is applied in the wavenumber domain for each image point where, $\mathbf{m}_{mig}(\mathbf{x})$ are taken as spatially varying 3D migrated image windows (their size is a function of the dominant local frequency/wavelength). The application of the LSM in the wavenumber domain also requires transforming the 3D windows $\mathbf{m}_{mig}(\mathbf{x})$ into the wavenumber domain. Equation 3 can be now written as,

$$\Theta(\hat{\mathbf{m}}) = \frac{1}{2} \left\| \boldsymbol{H}(\mathbf{k}_\nu) \hat{\mathbf{m}}(\mathbf{k}_\nu) - \mathbf{m}_{mig}(\mathbf{k}_\nu) \right\|^2 \ . \quad (7)$$

**Examples**

We implemented the described method on the Volve field OBC dataset. Figure 2 illustrates a structurally complex section at depth 1200m, before and after employing the LAD-PSF LSM operator. The PSF was generated from the LAD for each image point, using a 20Hz Ricker wavelet at the target area. The LSM image reveals enhanced details and high-dip features, improving the visibility of reflecting events and achieving clearer separation between subsurface elements. These effects are particularly evident when examining the wavenumber domain of the whole section. In this domain, we observe additional high-frequency content, increased exposure of higher-dipping wavenumbers, and improved amplitude balance among the events.

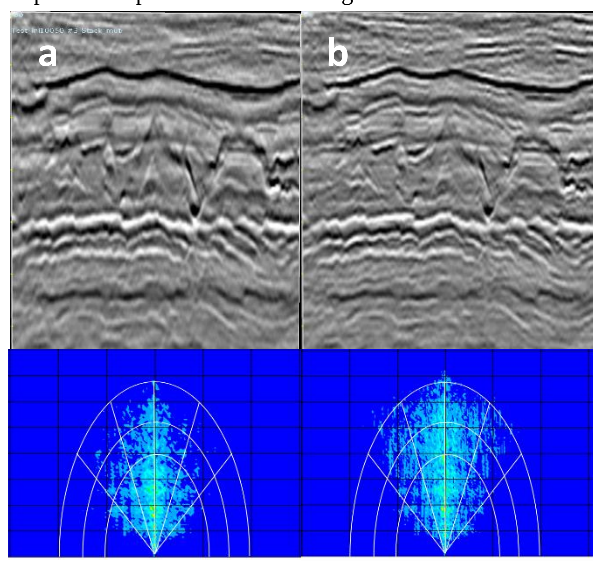


*Figure 2 Volve stack section before (a) and after (b) LAD - PSF LSM. The section is shown in the spatial domain (top) and wavenumber domain (bottom).*

The second example shown in Figure 3 demonstrates the implementation of the LAD-PSF LSM method on the Southern North Sea dataset. In this case the dominant frequency was computed adaptively for each image grid point, yielding a well-balanced deblurred image with enhanced resolution and better representation of the dipping reflecting events.

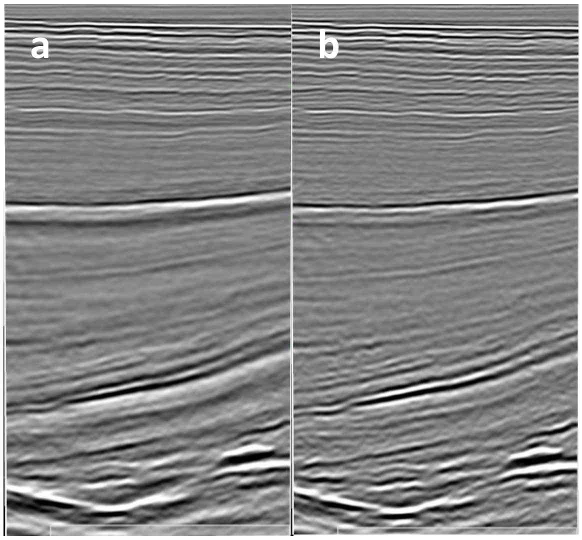


*Figure 3 Southern North Sea section before (a) and after (b) LAD - PSF LSM.*

**Conclusions**

This work presents an innovative, efficient LSM method where the high-resolution full-azimuth local angle domain (LAD) imaging system is used to compute high-resolution PSF operators directly at the fine image grid points. These PSFs are used to obtain high-resolution images. The method is applied for enhancing the quality and resolution of already existing migrated images. Note that the enhancement can be applied to reflection angle image data as well. We demonstrated the image enhancement capability, (de-blurring and higher resolution) of the proposed LAD-PSF LSM along two real data examples.

**Acknowledgments**

We thank Equinor and the Volve license partners for the Volve field dataset.