## Robust phase-driven time warping FWI for salt reconstruction

Guanghui Huang\*, Alejandro Alcudia-León, Jaime Ramos-Martínez and Nizar Chemingui, PGS

## Summary

We introduce a phase-driven Full Waveform Inversion (FWI) based on the Time Warping Extension (TWE) method. Our novel TWE objective function effectively addresses the challenging issue of cycle-skipping, often encountered in complex regions with uncertain salt geometry, particularly around salt flanks and bases. Additionally, it remains robust against amplitude mismatches arising from assumptions made in earth model representations.

We validate the effectiveness of our approach through a controlled experiment and successfully apply it to Ocean Bottom Node (OBN) data acquired in the Gulf of Mexico using ultra-long offsets. We demonstrate that our approach is capable of reshaping salt geometries, as evidenced by the improved imaging of subsalt structures.

#### Introduction

FWI for reconstructing salt and geobodies with large velocity contrasts compared to surrounding sediments requires special considerations in algorithm design. Even slight geometry errors can result in significant cycleskipping, a limitation not effectively addressed by conventional FWI algorithms based on the L2 objective function. Moreover, amplitude mismatches stemming from assumptions in earth model representations can lead to incorrect velocity updates. Several authors have addressed these issues by employing various cross-correlation objective functions (e.g., Lou et al., 2016; Choi and Alkhalifah, 2016; Zhang et al., 2018).

In this study, we developed a novel FWI approach based on the Time Warping Extension (TWE) technique, as detailed by Huang et al. (2021). The original objective function enables seamless transition from a time-shift application for overcoming cycle-skipping, to conventional least-squares FWI aimed at refining the velocity field. Our new approach prioritizes phase alignment between recorded and synthetic events. Unlike cross-correlation-based algorithms, our method automates the warping process, eliminating the need for window selection.

First, we describe the main distinction from the original approach presented by Huang et al. (2021). Then, we assess our new approach with a synthetic example inspired by a geological scenario in Brazil. Finally, we demonstrate its successful application to ultra-long offset OBN data acquired in the Gulf of Mexico.

## Theory

Huang et al. (2021) introduced an FWI method that uses time-warping as an extension domain to overcome cycleskipping. The warping function dynamically aligns the recorded field data with the synthetic data, while enforcing accurate representation of the physical time. Their original TWE objective function is

$$J[m,T] = \frac{1}{2\lambda} ||F[m] - d(T(t))||_2^2 + \frac{1}{2} ||t - T(t)||_2^2,$$
(1)

where u(t) = F(m), *m* is the velocity, T(t) is the timewarping extension that that dynamically aligns the field data *d* with the modeled data *u*. Both velocity and warping times are estimated simultaneously within a single optimization problem using the alternate direction method (ADM).

In this study, we assume that  $\lambda$  is sufficiently small, aiming to maximize the convexity of the objective function with respect to the time shift. Consequently, the inversion process is entirely guided by the phase alignment between the recorded and modeled data, as described in the second term of equation (1). It can be demonstrated that the adjoint source of the phase-driven TWE objective function can be approximated by

$$T(t) = (T(t) - t)\frac{\partial}{\partial t}u(t).$$
(2)

### Synthetic example

We demonstrate the effectiveness of our algorithm using a controlled experiment. In Figures 1a and 1b, we show the true velocity and density models used to compute the input data, resembling an OBN survey geometry. To assess the insensitivity of the algorithm to amplitude mismatch, the input data were computed with the variable-density acoustic wave-equation and the inversion was performed with a constant density modeling engine. The initial salt geometry (depicted in Figure 1c) significantly deviates from the true geometry. We compare the performance of our algorithm with conventional FWI based on the L2 objective function. To reconstruct the background model, we employ the velocity sensitivity kernel (Ramos-Martinez et al., 2016). Once the background is resolved, the high wavenumber components of the model can be incorporated using the conventional cross-correlation FWI gradient. Velocity updates obtained from the conventional L2 objective function (Figure 1e) confirm the inability of conventional algorithms to address the significant cycle-skipping associated with poor initial velocity models. In contrast, our

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algorithm yields velocity updates (Figure 1f) that closely resemble the true updates depicted in Figure 1d.

#### Field data example

We applied our algorithm to ultra-long offset OBN data acquired in the Gulf of Mexico. The survey design enabled recording of maximum inline offsets up to approximately 40 km and minimum crossline offsets of around 12 km. The hydrophone data underwent preconditioning to improve the signal-to-noise ratio at lower frequencies, enabling initiation of our FWI workflow at approximately 2.5 Hz. Our workflow strategy focused on leveraging low-frequency refracted waves to update the background sediment model and reconstruct the salt geometry. After sufficient iterations to enhance the background velocity model, we exploited the full wavefield to further refine the background and enhance the resolution of the inverted model. Figures 2a and 2b illustrate an example of the initial velocity model and the 4 Hz TWE FWI model for a crossline section, respectively, showcasing successful reshaping of the salt geometry driven entirely by TWE-FWI utilizing refractions and reflections. Figures 2c and 2d show an example of the initial velocity model and the 4 Hz FWI model for a depth slice, respectively, highlighting the increase in model resolution and definition of sediments and salt boundaries. Seismic sections in Figures 2e and 2f depict 8 Hz RTM images generated for QC purposes using the initial velocity model and the 4 Hz TWE FWI model. The imaging improvement and definition of salt overhangs and the base of salt are notably enhanced as a result of model updates using the phase-driven TWE FWI approach.

#### Conclusions

We introduced an innovative phase-driven FWI approach tailored to repair salt geometry. Our FWI method, integrating a time-warping extension, directly tackles the issue of severe cycle-skipping prevalent in complex environments with high-contrast geobodies. Moreover, it exhibits robustness against amplitude mismatches between recorded and modeled data. Through an illustrative example in the Gulf of Mexico, we showcased the effectiveness of our approach in reconstructing salt overhangs. Our results underscore significant enhancements in imaging resolution, particularly beneath salt and at intricate salt boundaries.

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Figure 1: Synthetic example illustrating a representative offshore Brazil setting. True models of a) velocity and b) density; c) initial velocity model and d) true difference relative to the true model. Velocity updates obtained with the conventional e) L2 objective function and f) our phase-driven TWE objective function.



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Figure 1: Field data example from the Gulf of Mexico. Initial velocity model for a) crossline section and c) depth slice; b) 4 Hz TWE FWI model for b) crossline section and d) depth slice; e) 8 Hz QC RTM with initial velocity; f) 8 Hz QC RTM with 4 Hz TWE FWI model.